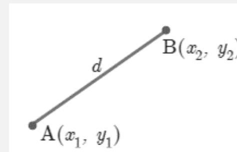
5. Coordinate Geometry

Introduction for Exercise 5.1

Concept corner

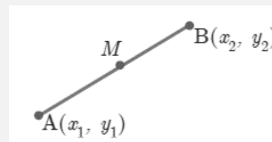
Distance between two points:



Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$|AB| = d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

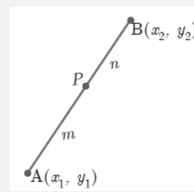
Mid – point of line segment:



The mid – point M, of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Section Formula

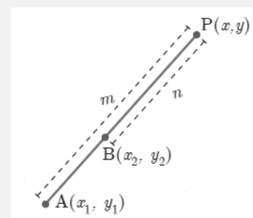
Internal Division:



Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two distinct points such that point $P(x, y)$ divides AB **internally** in the ratio $m{:}\,n$.

Then the coordinates of P are given by $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

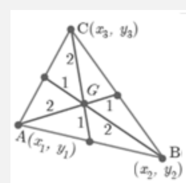
External Division:



Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two distinct points such that point $P(x, y)$ divides AB **externally** in the ratio $m{:}\,n$.

Then the coordinates of P are given by $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$.

Centroid of a triangle:



The coordinates of the centroid (G) of a triangle with vertices $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are given by $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

Area of a Triangle:

Area of $\Delta ABC = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$ Sq. units.

Another form: Area of $\Delta ABC = \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$ sq.units

$= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\}$ sq.units

Note: "As the area of a triangle can never be negative, we must take the absolute value, in case are happens to be negative".

Collinearity of three points:

Three points $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ will be collinear if the area of $\Delta ABC = 0$.

Note: Another condition for collinearity:

If $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear points, then

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$ or $x_1y_2 + x_2y_3 + x_3y_1 = x_1y_3 + x_2y_1 + x_3y_2$.

Area of the quadrilateral:

Area of the quadrilateral $ABCD$

$$= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4)\} \text{ sq.units}$$

Note:

- To find the area of a quadrilateral, we divide it into triangular regions, which have no common area and then add the area of these regions.
- The area of the quadrilateral is never negative. That is, we always take the area of quadrilateral as positive.

Introduction for Exercise 5.2

Concept corner

Note: The inclination of a line or the **angle of inclination** of a line is the angle which a straight line makes with the positive direction of X axis measured in the counter – clockwise direction to the part of the line above the X axis. The inclination of the line is usually denoted by θ.

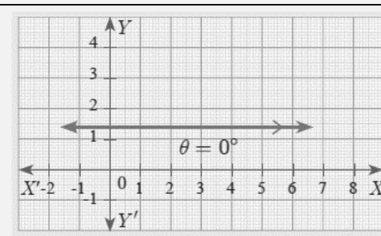
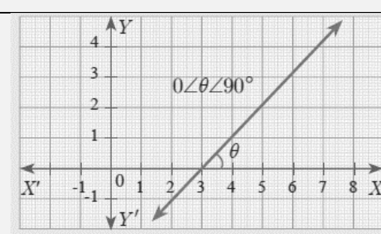
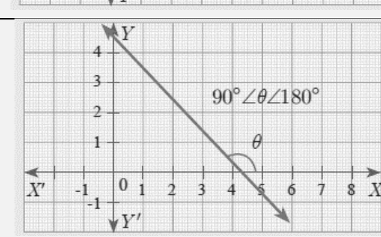
- The inclination of X axis and every line parallel to X axis is 0°
- The inclination of Y axis and every line parallel to Y axis is 90°

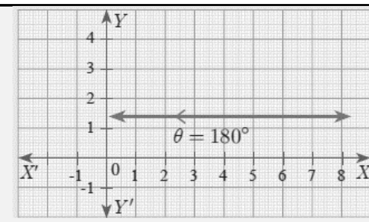
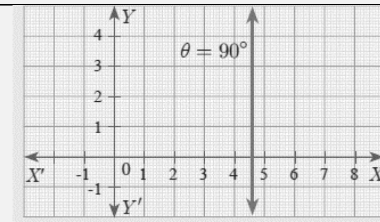
Definition: If θ is the angle of inclination of a non-vertical straight line, then $tan\theta$ is called the **slope** or gradient of the line and is denoted by m.

Therefore the slope of the straight line is $m = tan\theta,\ 0 \le \theta \le 180^\circ, \theta \ne 90^\circ$

Note: The slope of a vertical line is undefined.

Values of slopes

S.no	Condition	Slope	Diagram
(i)	$\theta = 0^\circ$	The line is parallel to the positive direction of X axis.	
(ii)	$0 < \theta < 90^\circ$	The line has positive slope (A line with positive slope rises from left to right)	
(iii)	$90^\circ < \theta < 180^\circ$	The line has negative slope (A line with negative slope falls from left to right).	

(iv)	$\theta = 180^{\circ}$	The line is parallel to the negative direction of X axis.	
(v)	$\theta = 90^{\circ}$	The slope is undefined.	

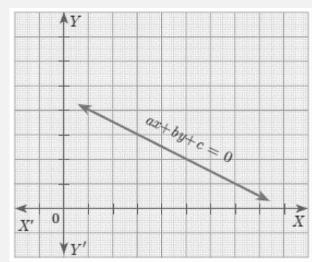
- Two non-vertical lines are parallel if and only if their slopes are equal.
- Two non-vertical lines with slopes m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$.

Note:

- Let l_1 and l_2 be two lines with well-defined slopes m_1 and m_2 respectively, then
  (i) l_1 is parallel to l_2 if and only if $m_1 = m_2$. (ii) l_1 is perpendicular to l_2 if and only if $m_1 m_2 = -1$.
- In any triangle, exterior angle is equal to sum of the interior opposite angles.
- If the slopes of both the pairs of opposite sides are equal then the quadrilateral is a parallelogram.

Introduction for Exercise 5.3

Concept corner

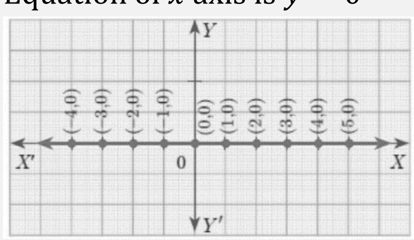


Straight line: Any first degree equation in two variables x and y of the form $ax + by + c = 0$ where a, b, c are real numbers and at least one of a, b is non-zero is called "**Straight line**" in xy plane.

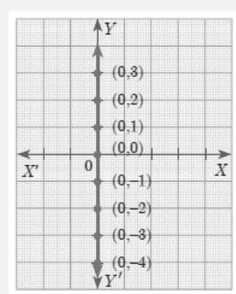
Equation of coordinate axes

The X axis and Y axis together are called coordinate axes.

Equation of x axis is $y = 0$



Equation of y axis is $x = 0$

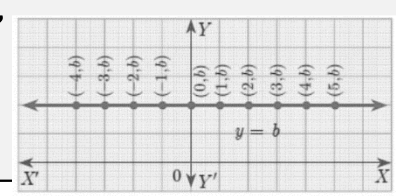


Equation of a straight line parallel to X axis

Let AB be a straight line parallel to X axis, which is at a distance 'b'

Then y coordinate of every point on 'AB' is 'b'.

Therefore, the equation of AB is $y = b$

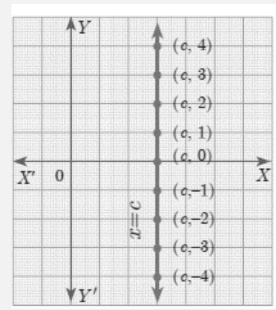


Note:

- If $b > 0$, then the line $y = b$ lies above the X axis
- If $b < 0$, then the line $y = b$ lies below the X axis
- If $b = 0$, then the line $y = b$ is the X axis itself.

Equations of a Straight line parallel to the Y axis

Let CD be a straight line parallel to Y axis, which is at a distance $'c'$. Then x coordinate of every point on CD is $'c'$. The equation of CD is $x = c$.



Note:

- If $c > 0$, then the line $x = c$ lies right to the side of the Y axis
- If $c < 0$, then the line $x = c$ lies left to the side of the Y axis
- If $c = 0$, then the line $x = c$ is the Y axis itself.

Slope – Intercept Form

Every straight line that is not vertical will cut the Y axis at a single point. The y coordinate of this point is called **y intercept** of the line.

A line with slope m and y intercept c can be expressed through the equation $y = mx + c$

- If a line with slope $m, m \neq 0$ makes x intercept d, then the equation of the straight line is $y = m(x - d)$.
- $y = mx$ represent equation of a line with slope m and passing through the origin.

Equation of Straight line in various forms:

	Name	Form
1	General form	$ax + by + c = 0$
2	Point – slope form	$y - y_1 = m(x - x_1)$
3	Slope - intercept	$y = mx + c$
4	Two point form	$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

	Name	Form
5	Intercept form	$\frac{x}{a} + \frac{y}{b} = 1$
6	Parallel to Y axis	$x = c$
7	Parallel to X axis	$y = b$

Introduction for Exercise 5.4

Concept corner

General Form of a Straight Line

The linear equation (first degree polynomial in two variables x and y) $ax + by + c = 0$ (where a, b and c are real numbers such that at least one of a, b is non-zero) always represents a straight line. This is the general form of a straight line.

Now, let us find out the equations of a straight line in the following cases

(i) parallel to $ax + by + c = 0$

- The equation of all lines parallel to the line $ax + by + c = 0$ can be put in the form $\boldsymbol{ax + by + k = 0}$ for different values of k.

(ii) perpendicular to $ax + by + c = 0$

- The equation of all lines perpendicular to the line $ax + by + c = 0$ can be written as $\boldsymbol{bx - ay + k = 0}$ for different values of k.

(iii) The point of intersection of two intersecting straight lines

- Two straight lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ where the coefficients are non-zero, are

(i) parallel if and only if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$; That is, $a_1b_2 - a_2b_1 = 0$

(ii) perpendicular if and only if $a_1a_2 + b_1b_2 = 0$

Slope of a straight line $ax + by + c = 0$:

Slope $m = \frac{-\text{ coefficient of } x}{\text{coefficient of } y} = -\frac{a}{b}$, y intercept $= \frac{-\text{ constant term}}{\text{coefficient of } y} = -\frac{c}{b}$